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Singular behavior of the stress field at the wedge-shaped corners of branching cracks

V. K. VARATHARAJULU

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York 14853, U.S.A.

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SUMMARY
The method of homogeneous solutions using the self-similarity nature of certain field variables is employed to investigate the behavior of the stress field in the vicinity of wedge-shaped corners of branching cracks, a problem which poses difficulty when attempted by other methods. Closed form solutions are obtained by using the theory of complex variables. Two examples are studied; 1) when an existing crack branches off with constant velocity under an arbitrary angle and 2) when an existing crack bifurcates with different velocities at arbitrary angles. The method presented here can also be extended to study the stress field behavior at the wedge-shaped corners created by any number of branching cracks.

1. Introduction

When a crack branches from its plane of propagation, additional singularities are generated at the wedge-shaped corners where the crack changes its direction of propagation in addition to the singularities at the crack tips. The study of these singularities is important in the investigation of the possibility of further fracture at the juncture of the main crack and the branching crack. For the elastostatic case, the stress behavior at the corners of branching cracks has been studied by Sih [1] employing the method of eigen function expansion. The method is quite well known and we refer to Williams [2, 3]. However, when the crack propagates at high velocity under dynamic loading conditions, inertia effects play an important role in the analysis and the associated elastodynamic problem poses difficulty when attempted by standard methods, such as transform techniques etc. This paper presents an efficient way of computing these singularities by employing the method of homogeneous solutions.

The method of homogeneous solutions has been extensively used in wave propagation problems and crack propagation problems (see, for example [4–8]). The method is briefly discussed in [9]. The method of solution is based on the observation that certain field variables are self-similar, i.e., they depend on r/t and θ rather than on r, t and θ separately. An important step in the analysis is the use of Chaplygin's transformation, which reduces the problem to the solution of Laplace's equation in a semi-infinite strip containing slits. The conformal mapping technique is subsequently employed to map the semi-infinite strip on a half plane. The appropriate harmonic function in the half plane is obtained by using elements of the theory of functions of complex variables. The singular behavior of the stress field at the wedge-shaped corners is finally obtained by a limiting process.

Since certain features of the in-plane and the anti-plane problems are similar, the mathematically simpler anti-plane shear problem is studied here which might greatly aid the study of the in-plane problem.

2. The diffraction problem

We consider an isotropic, homogeneous, linearly elastic medium containing a semi-infinite crack. At time $t = 0$, a plane incident wave strikes the tip of the crack ($x = y = 0$) and is of the form

$$w_{\text{inc}}(x, y, t) = f(\tau) = f[t + (x/c)\sin\alpha - (y/c)\cos\alpha], \tag{2.1}$$

where w is the out-of-plane displacement, α is the angle the wave front makes with the x-axis, c is the velocity of the transverse waves and $f(\tau) \equiv 0$ for $\tau \leqq 0$. Subsequently the wave is reflected and diffracted and the wave patterns are shown in Fig. 1. It is assumed that cracks emanate from the crack tip ($x = y = 0$) under arbitrary angles $\pi\kappa_j$ ($j = 1, 2, .., n$) and at constant velocities v_j ($v_j/c < 1$), creating wedge-shaped corners, at the instant the crack tip is struck by the wave. These corners are denoted by P_l ($l = 1, 2, \ldots, n+1$), see Fig. 1. In Fig. 1, the κ_j are taken such that $-1 < \kappa_1, \kappa_2, \ldots, \kappa_m < 0$ and $0 < \kappa_{m+1}, \ldots, \kappa_n < 1$.

The reflection and diffraction of the incident wave involve antiplane motions only, which are governed by

$$\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 = (1/c^2)\partial^2 w/\partial t^2 \tag{2.2}$$

where

$$c = (\mu/\rho)^{\frac{1}{2}}. \tag{2.3}$$

In eq. (2.3), μ and ρ are the shear modulus and mass density respectively. To investigate the cylindrical diffracted wave and the stress field near the wedge-shaped corners, it is convenient to express the Laplacian in polar coordinates with origin fixed at the crack

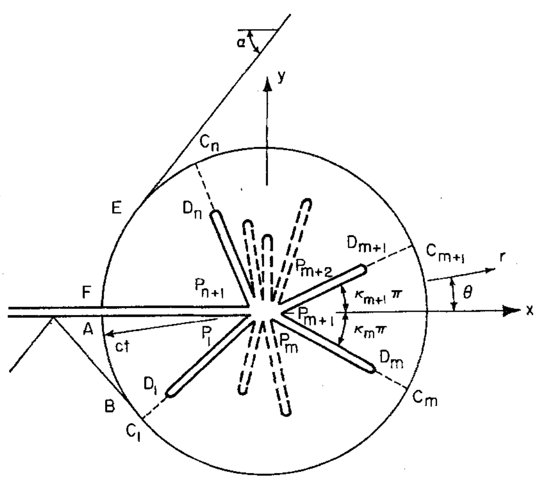


Figure 1. Pattern of wavefronts and positions of the crack tips.

tip ($x = y = 0$). Equation (2.2) is thus rewritten as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}=\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2}. \tag{2.4}$$

The problem at hand thus consists of finding solutions of eq. (2.4) satisfying certain boundary conditions on the cylindrical wavefront imposed by eq. (2.1) and the stress free condition on the crack surfaces.

3. Method of solution

The method of solution is based on the observation that certain field variables are self-similar, i.e., they depend on r/t and θ rather than on r, t and θ separately. It can be easily shown that for an incident step displacement wave, the displacement w is self-similar, see [10], where as for an incident step stress wave, the particle velocity

$$\dot{w} = \partial w/\partial t \tag{3.1}$$

is self-similar, see [6, 7, 8]. Since stress wave propagation is more relevant for practical purposes, we use the self-similarity nature of the particle velocity. We now proceed to solve the problem.

Introducing a new variable

$$s = r/t \tag{3.2}$$

the equation for $\dot{w}(s, \theta)$ follows from (2.4) as

$$s^2\left(1-\frac{s^2}{c^2}\right)\frac{\partial^2 \dot{w}}{\partial s^2}+s\left(1-2\frac{s^2}{c^2}\right)\frac{\partial \dot{w}}{\partial s}+\frac{\partial^2 \dot{w}}{\partial \theta^2}=0. \tag{3.3}$$

It can be shown that eq. (3.3) is elliptic for $s \leqq c$ and hyperbolic for $s > c$ (see, for example, [11]). Since we are interested to study the singularity in the vicinity of the wedge-shaped corners at the juncture of the branching cracks and main crack, it is enough to consider the case when $s \leqq c$, i.e., inside the cylindrical wave front.

For $s \leqq c$, the following transformation

$$\beta = \operatorname{arccosh}(c/s) \tag{3.4}$$

which is known as the Chaplygin's transformation, reduces eq. (3.3) to Laplace's equation

$$\partial^2\dot{w}/\partial\beta^2 + \partial^2\dot{w}/\partial\theta^2 = 0. \tag{3.5}$$

The real transformation (3.4) maps the interior of the circular domain into a semi-infinite strip containing slits in the $\beta-\theta$ plane, see Fig. 2, where corresponding points are indicated. An interesting aspect of this transformation is that radial lines in the $r-\theta$ plane become horizontal lines in the $\beta-\theta$ plane. The boundary conditions on the surface of the cracks and the cylindrical wave front must also be transformed in the $\beta-\theta$ plane.

Within the semi-infinite strip in the $\beta-\theta$ plane the solution of Laplace's equation may be written as the real part of an analytic function $G(\gamma)$

$$\dot{w} = \operatorname{Re}[G(\gamma)] \tag{3.6}$$

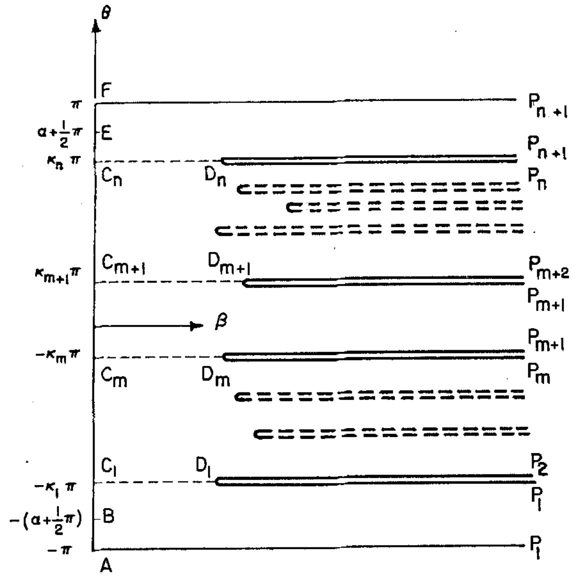


Figure 2. Domain in the $\theta - \beta$ plane.

where

$$\gamma = \beta + i\theta. \tag{3.7}$$

It is usually difficult to determine the function $G(\gamma)$ which satisfies the boundary conditions in the γ-plane. However, the function $G(\gamma)$ can, in principle, be obtained by the use of conformal mapping (in our case we use the Schwarz–Christoffel transformation)

$$\gamma = \omega(\zeta) = C_1 \int_1^{\zeta} \frac{(u - \kappa_{m+1}) \prod_{j=1}^{m} (u + \xi_j^D) \prod_{j=m+2}^{n} (u - \xi_j^D)}{(1 - u^2)^{\frac{1}{2}} \prod_{l=1}^{m} (u + \xi_l^P) \prod_{l=m+1}^{n+1} (u - \xi_l^P)} du + C_2 \tag{3.8}$$

where

$$\zeta = \xi + i\eta \tag{3.9}$$

which maps the γ-plane onto the upper half of the ζ-plane. Here ξ_j^D $(j \neq m+1)$ and ξ_l^P define the points in the ζ-plane corresponding to D_j $(j \neq m+1)$ and P_l. C_1 and C_2 are complex constants. The ζ-plane is shown in Fig. 3, where ξ_B and ξ_E define the points

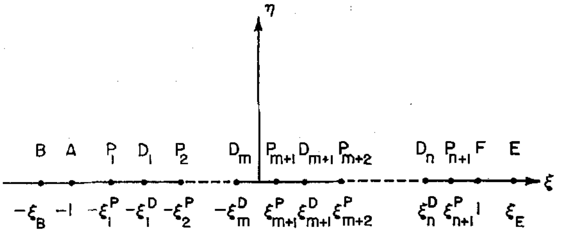


Figure 3. Mapping on the ζ-plane.

corresponding to B and E in Fig. 1. In eq. (3.8), C_1, C_2, ξ_l^P and ξ_j^D ($j \neq m + 1$) are unknowns. The mapping of the point F gives $C_2 = i\pi$. By examining changes of the imaginary parts at P_l and comparing the coordinates of the points D_j in the γ and ζ-planes, we obtain a system of non-linear equations, which allow us to compute the remaining unknowns for given values of κ_j and v_j/c. In the ζ-plane the solution to the corresponding problem can be obtained by the method of sectionally holomorphic function. This method has been discussed in great detail by Muskhelishvili [12]. The result is of the form

$$\dot{w} = \mathrm{Re}[F(\zeta)]. \tag{3.10}$$

The relevant stress field should then be obtained inside the cylindrical wavefront.

4. Stress field in vicinity of wedge-shaped corners

To study the singular behavior of the stress field in the vicinity of wedge-shaped corners, the shear stresses must be determined. The only non-vanishing stresses in the anti-plane case are given by

$$\tau_{\theta z} = (\mu/r)\partial w/\partial\theta \tag{4.1}$$

and

$$\tau_{rz} = \mu\,\partial w/\partial r. \tag{4.2}$$

It can be easily checked that the derivatives of these stresses with respect to time employing eqs. (3.4) and (3.10) can be written as

$$\dot{\tau}_{\theta z} = \frac{\mu}{r}\,\mathrm{Re}\left(\frac{\partial F}{\partial\zeta}\,\frac{\partial\zeta}{\partial\theta}\right) \tag{4.3}$$

and

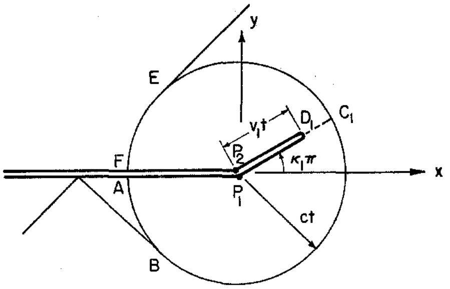
$$\dot{\tau}_{rz} = \mathrm{Re}\left(\frac{\partial F}{\partial\zeta}\,\frac{\partial\zeta}{\partial\beta}\,\frac{\partial\beta}{\partial r}\right). \tag{4.4}$$

To obtain the expressions for these stresses in the $r - \theta$ plane, ζ has to be solved in terms of γ from eq. (3.8). The singular behavior of the stress field at the wedge-shaped corners can then be obtained by letting $r \to 0$. The procedure, however, is very difficult because of the complexity of the mapping function (3.8) involved. A limiting process which will overcome this difficulty is shown in the sequel by considering some examples when the incident wave is a step stress wave defined by

$$w_{\mathrm{inc}}(x, y, t) = W_0 \tau H(\tau) \tag{4.5}$$

where W_0 is a constant, τ is defined in eq. (2.1) and $H(\tau)$ is the Heaviside step function.

Example 1: Let us assume that the semi-infinite crack propagates with constant velocity v_1 at an angle $\kappa_1\pi$ at the instant the crack tip is struck by the wave. This corresponds to the case when $n = 1$ in Section 3 and is known as *skew crack propagation*, see [6]. There are two wedge-shaped corners that are denoted by P_1 and P_2, see Fig. 4. For a step stress



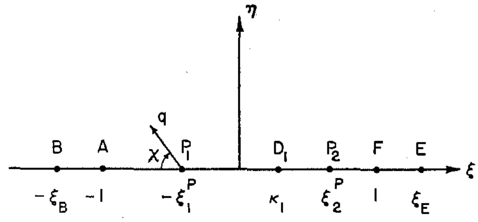


Figure 4. Skew crack propagation.
Figure 5. Local polar coordinates at point P_1 in the ζ-plane-skew crack propagation.

incident wave of the form given by eq. (4.5), the boundary conditions in terms of s and θ inside the cylindrical wave front can be written as, see [6]

$$\theta = -\pi; \quad s \leqq c\colon \partial\dot{w}/\partial\theta = 0 \tag{4.6}$$

$$-\pi \leqq \theta < -(\alpha + \pi/2); \quad s = c\colon \dot{w} = 2W_0 \tag{4.7}$$

$$-(\alpha + \pi/2) < \theta < \alpha + \pi/2; \quad s = c\colon \dot{w} = W_0 \tag{4.8}$$

$$\alpha + \pi/2 < \theta \leqq \pi; \quad s = c\colon \dot{w} = 0 \tag{4.9}$$

$$\theta = \pi; \quad s \leqq c\colon \partial\dot{w}/\partial\theta = 0 \tag{4.10}$$

$$\theta = \kappa_1\pi \pm \varepsilon; \quad s < v_1\colon \partial\dot{w}/\partial\theta = 0. \tag{4.11}$$

The γ-plane will be a semi-infinite strip with only one slit from $\beta_D = \operatorname{arccosh}(c/v_1)$ to infinity at $\theta = \kappa_1\pi$. The ζ-plane is shown in Fig. 5 where the corresponding points are indicated. The mapping function between γ- and ζ-planes is given by

$$\begin{aligned} \gamma = \omega(\zeta) = {} & (1 + \kappa_1)\{\ln[\{1 - (\xi_1^P)^2\}^{\frac{1}{2}}(1 - \zeta^2)^{\frac{1}{2}} + \zeta\xi_1^P + 1] - \ln(\zeta + \xi_1^P)\} \\ & + (1 - \kappa_1)\{\ln[\{1 - (\xi_2^P)^2\}^{\frac{1}{2}}(1 - \zeta^2)^{\frac{1}{2}} - \zeta\xi_2^P + 1] - \ln(\zeta - \xi_2^P)\} \\ & + i\pi \end{aligned} \tag{4.12}$$

where ξ_1^P and ξ_2^P are the image points of the wedge-shaped corners P_1 and P_2 in the ζ-plane.

The analytic function in the ζ-plane can easily be obtained, see [6, 13] and it is of the form

$$F'(\zeta) = \frac{i}{(\zeta^2 - 1)^{\frac{1}{2}}}\left[\frac{A}{\zeta + \xi_B^{(1)}} + \frac{B}{\zeta - \xi_E^{(1)}}\right] \tag{4.13}$$

where

$$A = -(W_0/\pi)[(\xi_B^{(1)})^2 - 1]^{\frac{1}{2}} \tag{4.14}$$

$$B = (W_0/\pi)[(\xi_E^{(1)})^2 - 1]^{\frac{1}{2}} \tag{4.15}$$

and prime denotes the differentiation with respect to the argument of the function. In eq. (4.13), $\xi_B^{(1)}$ and $\xi_E^{(1)}$ are the points in the ζ-plane corresponding to B and E in the $r - \theta$ plane and superscript (1) is used to differentiate from ξ_B and ξ_E in Section 3. The evaluation of $\xi_B^{(1)}$ and $\xi_E^{(1)}$ depends on the mapping function involved. The details of these calculations can be found in [13].

We will now investigate the nature of the stress field in the wedge-shaped corners P_1 and P_2. In the vicinity of $-\xi_1^P$, we write

$$\zeta \sim -\xi_1^P + \varepsilon_1^P \tag{4.16}$$

where ε_1^P is a complex number of small modulus. Keeping only the leading terms, the mapping function (4.12) can be written as

$$\gamma_1^P = \omega(\varepsilon_1^P) \sim -(1+\kappa_1)\ln \varepsilon_1^P + i\kappa_1\pi. \tag{4.17}$$

By defining a local coordinate system q, χ, z at the point $-\xi_1^P$ in the ζ-plane, see Fig. 5, we can express ε_1^P as

$$\varepsilon_1^P = q\exp[i(\pi-\chi)]. \tag{4.18}$$

Substituting eq. (4.18) in eq. (4.17) and separating the real and imaginary parts, we obtain

$$q = \exp[-\beta/(1+\kappa_1)]; \qquad \chi = (\pi+\theta)/(1+\kappa_1). \tag{4.19a, b}$$

In the vicinity of the point ξ_1^P, eq. (4.3) becomes

$$\dot{\tau}_{\theta z} = (\mu/r)\,\mathrm{Re}[iF'(\xi_1^P)/\omega'(\varepsilon_1^P)] \tag{4.20}$$

where $F'(\xi_1^P)$ follows from eq. (4.13) with ζ replaced by ξ_1^P.

Using a well-known representation for arccosh, we find from eqs. (3.2) and (3.4) that

$$\beta = \ln\left\{\frac{ct}{r} + \left[\left(\frac{ct}{r}\right)^2 - 1\right]^{\frac{1}{2}}\right\}. \tag{4.21}$$

For small values of r/ct, eq. (4.21) is subsequently employed in eq. (4.19) to write

$$q \sim \left(\frac{2ct}{r}\right)^{-1/(1+\kappa_1)}. \tag{4.22}$$

Differentiating eq. (4.17) with respect to ε_1^P and substituting in eq. (4.20) with q, χ and ε_1^P given by eqs. (4.22), (4.19b) and (4.18) respectively and subsequently integrating with respect to time, we find

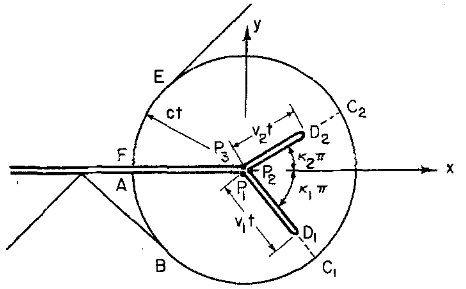
$$(\tau_{\theta z})_1^P \sim \frac{1}{2}\,\frac{\mu}{\kappa_1 c}\,F'(\xi_1^P)\left(\frac{2ct}{r}\right)^{\kappa_1/(1+\kappa_1)}\sin\frac{\pi+\theta}{1+\kappa_1}. \tag{4.23}$$

It should be noted that the stress is not singular if $\kappa_1 \leqq 0$. The general form of the dependence on r agrees with what is usually found at the vertex of a wedge, see [5].

A similar expression can also be obtained for the stress $\tau_{\theta z}$ in the vicinity of the point P_2 and is of the form

$$(\tau_{\theta z})_2^P \sim -\frac{1}{2}\,\frac{\mu}{\kappa_1 c}\,F'(\xi_2^P)\left(\frac{2ct}{r}\right)^{-\kappa_1/(1-\kappa_1)}\sin\frac{\pi-\theta}{1-\kappa_1}. \tag{4.24}$$

The stress remains finite for $\kappa_1 > 0$.



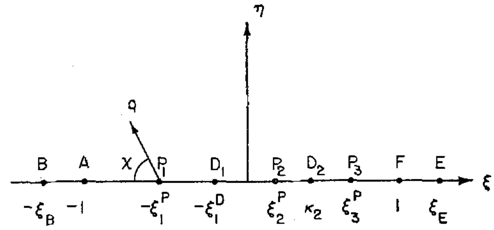


Figure 6. Crack bifurcation.
Figure 7. Local polar coordinates at the point P_1 – crack bifurcation.

Example 2: In this example, we consider the case when the semi-infinite crack divides into two branches, which propagate with different velocities v_1 and v_2 at arbitrary angles κ_1 and κ_2 respectively under the influence of the transient stress wave, see Fig. 6. Here ($n = 2$, $m = 1$) we have three wedge-shaped corners P_1, P_2 and P_3. This situation is known as *bifurcation*. The boundary conditions (4.6)–(4.10) remain essentially the same for this example also. However, instead of the condition (4.11), we have conditions which represent two stress free crack surfaces and they are:

$$\theta = -\kappa_1\pi \pm \varepsilon; \quad s < v_1: \; \partial\dot{w}/\partial\theta = 0, \tag{4.25}$$

$$\theta = \kappa_2\pi \pm \varepsilon; \quad s < v_2: \; \partial\dot{w}/\partial\theta = 0. \tag{4.26}$$

The γ-plane will now be a semi-infinite strip with two slits, one from $\beta_D^{(1)} = \operatorname{arccosh}(c/v_1)$ to infinity at $\theta = -\kappa_1\pi$ and the other one from $\beta_D^{(2)} = \operatorname{arccosh}(c/v_2)$ to infinity at $\theta = \kappa_2\pi$. The mapping function which maps the γ-plane to ζ-plane is of the form, see [13]:

$$\begin{aligned}\gamma = \omega(\zeta) = {} & (1-\kappa_1)\{\ln[\{1-(\xi_1^P)^2\}^{\frac{1}{2}}(1-\zeta^2)^{\frac{1}{2}} + \zeta\xi_1^P + 1] - \ln(\zeta + \xi_1^P)\} \\ & + (\kappa_1+\kappa_2)\{\ln[\{1-(\xi_2^P)^2\}^{\frac{1}{2}}(1-\zeta^2)^{\frac{1}{2}} - \zeta\xi_2^P + 1] - \ln(\zeta - \xi_2^P)\} \\ & + (1-\kappa_2)\{\ln[\{1-(\xi_3^P)^2\}^{\frac{1}{2}}(1-\zeta^2)^{\frac{1}{2}} - \zeta\xi_3^P + 1] - \ln(\zeta - \xi_3^P)\} \\ & + i\pi\end{aligned} \tag{4.27}$$

where ξ_1^P, ξ_2^P and ξ_3^P are the image points of the wedge-shaped corners P_1, P_2 and P_3 in the ζ-plane. The analytic function $F'(\zeta)$ is given by eq. (4.13) with $\xi_B^{(1)}$ and $\xi_E^{(1)}$ replaced by $\xi_B^{(2)}$ and $\xi_E^{(2)}$, where $\xi_B^{(2)}$ and $\xi_E^{(2)}$, the image points of B and E in the ζ-plane, are normally different from $\xi_B^{(1)}$ and $\xi_E^{(1)}$ because of the mapping function involved. The details of these calculations can be found in [13]. We will now proceed to study the stress $\tau_{\theta z}$ at the corners P_1, P_2 and P_3.

In the vicinity of the point ξ_1^P, i.e., $\zeta \sim -\xi_1^P + \varepsilon_1^P$, the mapping function (4.27) can be written, keeping only the leading terms, as

$$\gamma_1^P = \omega(\xi_1^P) \sim -(1-\kappa_1)\ln\varepsilon_1^P - i\kappa_1\pi. \tag{4.28}$$

With a local coordinate system q, χ at ξ_1^P in the ζ-plane, we can express ε_1^P as given by eq. (4.18). Substituting (4.18) in (4.28) and separating the real and imaginary parts, we obtain

$$q = \exp[-\beta/(1-\kappa_1)]; \qquad \chi = (\pi+\theta)/(1-\kappa_1). \tag{4.29a, b}$$

As expressed before, in the vicinity of the corners, we can write

$$(\dot{\tau}_{\theta z})_j^P = (\mu/r)\,\mathrm{Re}\,[iF'(\xi_j^P)/\omega'(\xi_j^P)], \quad j = 1, 2, 3. \tag{4.30}$$

Employing eq. (3.4) in (4.29a) and subsequently substituting in eq. (4.30) and integrating with respect to time, we obtain

$$(\tau_{\theta z})_1^P \sim -\frac{1}{2}\frac{\mu}{\kappa_1 c}F'(\xi_1^P)\left(\frac{2ct}{r}\right)^{-\kappa_1/(1-\kappa_1)}\sin\frac{\pi+\theta}{1-\kappa_1}. \tag{4.31}$$

In the vicinity of the points ξ_2^P and ξ_3^P, we take $\zeta \sim \xi_2^P + \varepsilon_2^P$ and $\zeta \sim \xi_3^P + \varepsilon_3^P$ respectively. Using these approximations in the mapping function (4.27), the shear stresses in the vicinity of the wedge-shaped corners P_2 and P_3 can be computed as before and they are given by

$$(\tau_{\theta z})_2^P \sim \frac{1}{2}\frac{\mu}{[1-(\kappa_1+\kappa_2)]c}F'(\xi_2^P)\left(\frac{2ct}{r}\right)^{-[1-(\kappa_1+\kappa_2)]/(\kappa_1+\kappa_2)}\sin\frac{\theta+\kappa_1\pi}{\kappa_1+\kappa_2} \tag{4.32}$$

and

$$(\tau_{\theta z})_3^P \sim -\frac{1}{2}\frac{\mu}{\kappa_2 c}F'(\xi_3^P)\left(\frac{2ct}{r}\right)^{-\kappa_2/(1-\kappa_2)}\sin\frac{\pi-\theta}{1-\kappa_2}. \tag{4.33}$$

It should be noted that $(\tau_{\theta z})_2^P$ remains finite for $(\kappa_1+\kappa_2) \geqq 0$ while $(\tau_{\theta z})_3^P$ remain finite for $\kappa_2 \geqq 0$.

In conclusion, it should be remarked that this method can be extended to n propagating cracks which give rise to $(n+1)$ wedge-shaped corners. For each corner, the mapping function should be expressed by keeping only the leading term.

Acknowledgments

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REFERENCES

[1] G. C. Sih, Stress distribution near internal crack tips for longitudinal shear problems, *Journal of Applied Mechanics*, 32 (1965) 51–58.

[2] M. L. Williams, Stress singularities resulting from various boundary conditions in angular corners of plates in extension, *Journal of Applied Mechanics*, 74 (1952) 526–528.

[3] M. L. Williams, Surface stress singularities resulting from various boundary conditions in angular corners of plates under Bending, *Proceedings of the First U.S. National Congress of Applied Mechanics*, (1952) 325–329.

[4] J. W. Miles, Homogeneous solutions in elastic wave propagation, *Quarterly of Applied Mathematics*, 18 (1959) 37–59.

[5] J. D. Achenbach, Shear waves in an elastic wedge, *International Journal of Solids and Structures*, 6 (1970) 379–388.
[6] J. D. Achenbach and V. K. Varatharajulu, Skew crack propagation at the diffraction of a transient stress wave, *Quarterly of Applied Mathematics*, 32 (1974) 123–135.
[7] V. K. Varatharajulu, Elastodynamic analysis of crack emanating from the vertex of a wedge, *Journal of Engineering Mathematics*, 8 (1974) 281–290.
[8] V. K. Varatharajulu, Dynamic analysis of wedge singularity in anti-plane shear, *Bulletin of Calcutta Mathematical Society*, to appear.
[9] J. D. Achenbach, *Wave propagation in elastic solids*, North-Holland Publishing Company, Amsterdam (1973) 154–160.
[10] J. D. Achenbach, Crack propagation generated by a horizontally polarized shear wave, *Journal of the Mechanics and Physics of Solids*, 18 (1970) 245–259.
[11] J. Irving and N. Mullineux, *Mathematics in physics and engineering*, Academic Press, New York (1959).
[12] N. I. Muskhelishvili, *Some basic problems of the mathematical theory of elasticity* (translated by J. R. M. Radok), Noordhoff, Groningen (1963).
[13] V. K. Varatharajulu, *Problems of elastodynamic crack propagation*, Doctoral Dissertation, Northwestern University, Evanston, Illinois (1973).